

Nuclear Potential with Two Pion Exchange

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We carefully calculate the nucleon-nucleon interaction due to two pion exchange processes by properly evaluating the corresponding Feynman diagrams. In the estimation, we have made no approximation, and instead, we carry out the numerical integrations of three Feynman parameters. It is found that the two pion exchange potential gives rise to the attractive force which corresponds to the effective scalar meson with its mass of $m_s \simeq 4.7m_\pi$ and its strength of $\frac{g_s^2}{4\pi} \simeq 1.45$. There is a strong isospin dependence of $(\tau_1 \cdot \tau_2)^2$ which cannot be simulated by the one boson exchange model calculations.

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I. INTRODUCTION

The structure of the nucleus can be described once the nucleon-nucleon interactions are properly known. Indeed there are already sufficiently large number of works available for the determination of the nucleon-nucleon potential [1–4]. The most popular nuclear interaction may be obtained by one boson exchange potential (OBEP) [5–7] where exchanged bosons are taken from experimental observations. In this case, the masses and the coupling constants of the exchanged bosons are determined from various methods, partly experimentally and partly theoretically. The discussions of the determination of these parameters may have some ambiguities, but one can see that the basic part of the nuclear force can be well understood until now.

However, there is one important problem which is not solved yet completely. This is related to the medium range attraction of the nucleon-nucleon potential, and this is normally simulated by the effective scalar meson exchange process. It is clear that there is no massive scalar meson in nature and, therefore, the artificial introduction of the scalar meson is indeed a theoretical defect of the one boson exchange model. This is indeed a homework problem for many years of nuclear physics research. However, this important problem is left unsolved for a long time since many of the nuclear theorists moved and got interested in the quark model calculations of the nucleon-nucleon interaction. By now, it becomes clear that the evaluation of the QCD based model has an intrinsic difficulty due to the gauge dependence of the quark color charge [8] and this strongly suggests that the meson exchange approach is indeed a right direction of the nuclear force calculations.

The origin of the medium range attraction has been discussed extensively, but until now there is no clear answer to this problem. Mostly, people believe that the medium range attraction may well be simulated by the second order calculation of one pion exchange potential in which the intermediate Δ -resonance state is included [9, 10]. However, it is also known that this cannot give rise to the sufficiently large contributions to the medium range attractions. Also, there are many calculations of the nucleon-nucleon interaction due to the two pion exchange processes [11–13], and this may indeed give rise to the medium range attraction even though until now there is no clear cut evaluation which can isolate the nuclear force contribution to the medium range attraction.

In this paper, we present a careful calculation of the two pion exchange processes, and in the evaluation, we have made no approximation. We first evaluate the corresponding Feynman diagrams which contain the integration of the four momentum k . This integration can be carried out exactly by introducing the Feynman

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parameters, and thus we are left with the integrations of the three Feynman parameters. This can be only carried out numerically, and we find that the corresponding T-matrix can be expressed reasonably well, at least, up to $q^2 \simeq (6m_\pi)^2$ as

$$T(q) \simeq -g_s^2(\tau_1 \cdot \tau_2)^2 \frac{1}{q^2 + m_s^2} \quad (1.1)$$

where

$$m_s \simeq 4.7m_\pi, \quad \frac{g_s^2}{4\pi} \simeq 1.45 \quad \text{with} \quad \frac{g_\pi^2}{4\pi} \simeq 8 \quad (1.2)$$

where the mass m_s and the coupling constant g_s should have a small increase (up to $\sim 20\%$) as the function of q^2 . Here, g_π and m_π denote the πNN coupling constant and pion mass, respectively. Therefore, the predicted values of the mass and the coupling constant of the effective σ meson from the two pion exchange diagrams are given for the $T = 0$ channel as

$$m_\sigma = m_s \simeq 650 \text{ MeV}, \quad \frac{g_\sigma^2}{4\pi} = 9 \times \frac{g_s^2}{4\pi} \simeq 13, \quad (T = 0, \text{ NN} - \text{state}) \quad (1.3)$$

which should be compared with the phenomenological values of the σ meson mass and coupling constant as determined by fitting to the nucleon-nucleon scattering data for the $T = 0$ channel [1, 2]

$$m_\sigma \simeq 615 \text{ MeV}, \quad \frac{g_\sigma^2}{4\pi} \simeq 11.7. \quad (1.4)$$

They indeed agree with each other at the quantitative level. The physical reason why the two pion exchange process becomes quite large can be easily understood. The one pion exchange potential is suppressed due to the pseudo-scalar interaction with nucleon, and thus the one pion exchange process in the second order diagram is relatively weak. Indeed, one knows that the one pion exchange calculation should pick up the small component in the nucleon Dirac wave function. However, the two pion exchange diagram has no such suppression and thus it can give rise to the largest contribution to the nucleon-nucleon potential. Since it is the fourth order process, it turns out to be an effective scalar interaction which is always attractive. In addition, it should be important to note that there is no double counting problem in this calculation since the two pion exchange process does not contain any one pion ladder type diagrams.

II. TWO PION EXCHANGE PROCESS

In addition to the one boson exchange processes, one should consider the two pion exchange diagrams in order to obtain a proper nucleon-nucleon interaction. There are of course some calculations of the two pion exchange diagrams [14], but until now there is no solid calculation of the two pion exchange potential which is compared to the observed data. However, one can easily convince oneself that the fourth order process involving the four γ_5 interactions is not suppressed at all, in contrast to the one pion exchange diagram where the γ_5 coupling is indeed suppressed by the factor of $\frac{m_\pi}{M}$ with M denoting the nucleon mass, which is basically due to the parity mismatch. Therefore, it should be very important to calculate the two pion exchange process properly in order to understand the medium range attraction of the nucleon-nucleon interaction.

Now, the evaluation of the two pion exchange Feynman diagram is done in a straight forward way [14, 15], and we find the corresponding T-matrix as

$$\begin{aligned} T = & ig_\pi^4(\tau_1 \cdot \tau_2)^2 \int \frac{d^4k}{(2\pi)^4} i\gamma_5^{(1)} \frac{1}{k^2 - m_\pi^2 + i\varepsilon} \frac{1}{(p_1 - k)^\mu \gamma_\mu^{(1)} - M + i\varepsilon} i\gamma_5^{(1)} \\ & \times i\gamma_5^{(2)} \frac{1}{(q - k)^2 - m_\pi^2 + i\varepsilon} \frac{1}{(p_2 + k)^\mu \gamma_\mu^{(2)} - M + i\varepsilon} i\gamma_5^{(2)} \end{aligned} \quad (2.1)$$

where p_1 (p'_1) and p_2 (p'_2) denote the initial (final) four momenta of the two nucleons, and q is the four momentum transfer which is defined as $q = p_1 - p'_1$. Here, we have ignored the crossed diagram which is much smaller than eq.(2.1). By noting

$$(\gamma_5^{(1)})^2 = 1, \quad (\gamma_5^{(2)})^2 = 1, \quad \gamma_5 \gamma^\mu = -\gamma^\mu \gamma_5$$

we can rewrite eq.(2.1) as

$$T = ig_\pi^4 (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)^2 \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2 - m_\pi^2} \frac{1}{(q - k)^2 - m_\pi^2} \times \frac{(p_1 - k)^\mu \gamma_\mu^{(1)} - M}{(p_1 - k)^2 - M^2} \times \frac{(p_2 + k)^\mu \gamma_\mu^{(2)} - M}{(p_2 + k)^2 - M^2}. \quad (2.2)$$

Now, we introduce the integration trick in terms of Feynman parameters x, y, z as

$$\frac{1}{abcd} = 6 \int_0^1 dx \int_0^x dy \int_0^y dz \frac{1}{[a + (b - a)x + (c - b)y + (d - c)z]^4}.$$

Further, we assume that the nucleons at the initial state are on the mass shell

$$(\not{p}_1 - M)u(p_1) = 0, \quad (\not{p}_2 - M)u(p_2) = 0$$

and therefore we also find

$$\bar{u}(p'_1) q^\mu \gamma_\mu u(p_1) = 0.$$

In addition, we take the non-relativistic limit for the nucleon motion and thus obtain

$$T \simeq -6ig_\pi^4 (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)^2 \int_0^1 dx \int_0^x dy \int_0^y dz \int \frac{d^4 k}{(2\pi)^4} \frac{\frac{1}{4}k^2 + M^2(2z - y)^2}{(k^2 - s)^4} \quad (2.3)$$

where s is defined as

$$s = q^2 ((y - x)^2 + y - x) + M^2(2z - y)^2 + m_\pi^2(1 - y). \quad (2.4)$$

The momentum integration of k can be easily done, and we find

$$T \simeq -\frac{g_\pi^4}{32\pi^2} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)^2 \int_0^1 dx \int_0^x dy \int_0^y dz \left[\frac{1}{s} - \frac{2M^2(2z - y)^2}{s^2} \right]. \quad (2.5)$$

This three dimensional integration of x, y, z can be done only numerically, and the calculated result can be well fit by the following shape

$$T \simeq -(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)^2 \frac{g_\pi^4}{32\pi^2} \times \frac{A}{\mathbf{q}^2 + m_s^2} \quad (2.6)$$

where A and m_s are found to be

$$A \simeq 0.57, \quad m_s \simeq 4.7m_\pi \simeq 650 \text{ MeV}.$$

Here, we replace the four momentum transfer of q^2 as

$$q^2 = q_0^2 - \mathbf{q}^2 \simeq -\mathbf{q}^2$$

since we may use the static approximation to a good accuracy

$$(q_0)^2 = \left(\sqrt{M^2 + \mathbf{p}_1^2} - \sqrt{M^2 + \mathbf{p}'_1{}^2} \right)^2 \simeq \frac{1}{4M^2} (\mathbf{p}_1^2 - \mathbf{p}'_1{}^2)^2 \ll \mathbf{q}^2. \quad (2.7)$$

If we take the value of the πNN coupling constant as $\frac{g_\pi^2}{4\pi} \simeq 8$, then we find

$$T \simeq -(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)^2 \frac{g_s^2}{q^2 + m_s^2} \quad (2.8)$$

where

$$\frac{g_s^2}{4\pi} \simeq \frac{1}{4\pi} \times \frac{g_\pi^4}{32\pi^2} \times 0.57 \simeq 1.45 \quad (2.9)$$

which are consistent with the values determined from the nucleon-nucleon scattering experiments.

It should be important to note that the present calculation suggests that the $T = 0$ channel of the nucleon-nucleon interaction is very strong in comparison with the $T = 1$ case. This means that the proton-neutron interaction is much stronger than one would naively expect. This fact is, of course, quite well known to nuclear structure physicists since they know from the fitting of the spectrum to experiments that the proton-neutron force in the residual nuclear interaction is quite strong while the interactions between identical nucleons are rather weak.

III. CONCLUSIONS

We have presented a new calculation of the old type Feynman diagram in the two pion exchange processes. The result is quite interesting since the corresponding T-matrix of the two pion exchange diagrams turns out to be just similar to the T-matrix of the effective σ meson exchange case where we obtain the corresponding effective σ meson mass $m_\sigma \simeq 650$ MeV and the effective coupling constant $\frac{g_\sigma^2}{4\pi} \simeq 13$ for $T = 0$ channel. These values of the mass and the coupling constant are found to agree with those values ($m_\sigma \simeq 615$ MeV, $\frac{g_\sigma^2}{4\pi} \simeq 11.7$) which are determined from the OBEP analysis of the nucleon-nucleon scattering experiments.

The result of the present study may invoke some further investigations of the nucleon-nucleon potential. In this calculation, we have employed the value of the πNN coupling constant g_π which is determined from the pion decay process into two photons, $\pi^0 \rightarrow \gamma + \gamma$, since its value is known to be $\frac{g_\pi^2}{4\pi} \simeq 8$ [16, 17]. On the other hand, people use the πNN coupling constant g_π in nucleon-nucleon potential which is somewhat larger than the above value, almost by a factor of two. This should be closely related to the mass and the coupling constant of the effective σ meson. Now, the property of the effective σ meson is determined by the two pion exchange process and thus we should reconsider the one boson exchange potential from the new point of view. It should be quite interesting to know to what extent we can understand the nucleon-nucleon scattering data with the new constraints of the πNN coupling constant. However, it is, at the same time, clear that the two pion exchange process cannot be simulated by the effective σ meson exchange as far as the isospin dependence is concerned.

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- [1] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987)
 - [2] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989)
 - [3] M. Lacombe, B. Loiseau, J.M. Richard, R. Vinh Mau, J. Cote, P. Pires, and R. de Tournreil, Phys. Rev. **C 21**, 861 (1980)
 - [4] M. Lacombe, B. Loiseau, J.M. Richard, R. Vinh Mau, J. Cote, P. Pires, and R. de Tournreil, Phys. Lett. **101B**, 139 (1981)
 - [5] D. Kiang, M.A. Preston and P. Tip, Phys. Rev. **170**, 907 (1968)
 - [6] J.J. Sakurai, "Advanced Quantum Mechanics", (Addison-Wesley, 1967)
 - [7] A. Bohr and B.R. Mottelson, "Nuclear Structure" (Vol. 1), (World Scientific, 1998)
 - [8] T. Fujita, "Symmetry and Its Breaking in Quantum Field Theory", (2nd edition, Nova Science Publishers, 2011)
 - [9] K. Holinde, Phys. Rep. **68**, 121 (1981)
 - [10] M.P. Valderrama and E.R. Ariola, Phys. Rev. **C83**, 044002 (2011)
 - [11] W.R. Wortman, Phys. Rev. **176**, 1762 (1968)

- [12] R.D. Haracz and R.D. Sharma, Phys. Rev. **176**, 2013 (1968)
- [13] M.H. Partovi and E.L. Lomon, Phys. Rev. **D2**, 1999 (1970)
- [14] F. Gross, "Relativistic quantum mechanics and field theory", (John Wiley & Sons, 1993)
- [15] J.D. Bjorken and S.D. Drell, "Relativistic Quantum Mechanics", (McGraw-Hill Book Company, 1964)
- [16] K. Nishijima, "Fields and Particles", (W.A. Benjamin, INC, 1969)
- [17] N. Kanda, R. Abe, T. Fujita and H. Tsuda, " Z^0 decay into two photons", arXiv:1109.0926v1 [hep-ph]